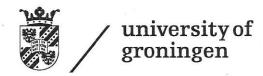
Midterm Exam Calculus 2

17 March 2017, 9:00-11:00



The midterm exam consists of 4 problems. You have 120 minutes to answer the questions. You can achieve 100 points which includes a bonus of 10 points.

1. [10+10+5 Points] Let the function $f: \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x,y) = \begin{cases} \frac{x^2y^2}{x^4+y^4} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}.$$

- (a) Is f continuous at (x, y) = (0, 0)? Justify your answer.
- (b) Use the definition of directional derivatives to determine for which unit vectors $\mathbf{u} = (v, w) \in \mathbb{R}^2$ the directional derivative $D_{\mathbf{u}} f(0, 0)$ exists.
- (c) Is f differentiable at (x, y) = (0, 0)? Justify your answer.
- 2. [10+10 Points] Consider the curve parametrized by $\mathbf{r}:[0,2\pi]\to\mathbb{R}^3$ with

$$\mathbf{r}(t) = (\sin t - t\cos t)\mathbf{i} + 2\mathbf{j} + (\cos t + t\sin t)\mathbf{k}.$$

- (a) Determine the parametrization by arc length.
- (b) At each point on the curve, determine the curvature of the curve.
- 3. [5+10+10 Points] Consider the ellipsoid

$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 3$$

which contains the point $(x_0, y_0, z_0) = (1, 2, 3)$.

- (a) Determine the tangent plane of the ellipsoid at the point (x_0, y_0, z_0) .
- (b) Show that near the point (x_0, y_0, z_0) the ellipsoid is locally given as the graph of a function over the (x, y) plane, i.e. there is a function $f: (x, y) \mapsto z$ such that near (x_0, y_0, z_0) the ellipsoid is locally given by z = f(x, y). Compute the partial derivatives $f_x(x_0, y_0)$ and $f_y(x_0, y_0)$ and show that the graph of the linearization of f at (x_0, y_0) agrees with the tangent plane found in part (a).
- (c) Use the method of Lagrange multipliers to find the points closest to and farthest away from the origin.
- 4. [20 Points] Determine

$$\iiint_W (2 + \sqrt{x^2 + y^2}) \, \mathrm{d}V,$$

where

$$W = \{(x, y, z) \in \mathbb{R}^3 | \sqrt{x^2 + y^2} \le \frac{z}{2} \le 3\}.$$